“Certainly the large scale econometric models and complicated VARs [...] have not been very successful in explanation and prediction. Thus it appears useful to start with a well understood, sophisticatedly simple model and check its performance empirically in explanation and prediction.”

Arnold Zellner

Abstract

The main goal of this paper is to illustrate how simple macroeconomic models can be used in different areas of applied research, and also as tools that can provide useful insights both at the more aggregate level (macro-policies designed to promote development), and in the decision-making process in financial markets. The models examined here are applied to two very different economies (United States and Brazil) with very satisfactory results in terms of simulation and prediction. In the paper, some basic issues in Economic Development are discussed and an overview of Portfolio Theory is presented.

Key words: Macroeconomic Modelling; Applied Econometrics; Economic Development; Portfolio Theory and Risk Analysis.

1. Introduction

This paper deals mainly with macro-econometric models used for purposes of simulation and forecasting, or simply “macro-models”, and their potential application in seemingly different, and certainly very relevant, areas. The models examined here are simple in their structure, and in fact two of them are standard material in Macroeconomics (Keynes model) and Econometrics (Kalecki-Klein model). It is interesting to realize – perhaps “disturbing” is more appropriate – that in the Macroeconomics field the potential use of basic models as tools of applied research is normally disregarded, and that in the area of Econometrics the simultaneous-equations models are rarely considered in their full economic-analytical content. Therefore a secondary objective in this paper is to explore paths that could lead to bridges over these academic-research “canyons”.

One may feel that these two areas examined as potential candidates for the application of macro-models have virtually nothing in common. Yet this observation is far from being accurate. One of the main trends of the global community which has evolved in the last few decades is that many elements which are central in the field of Economic Development – like the availability of adequate funding for both private and public investment projects – depend closely on the workings of financial markets, especially those in which longer-term securities are issued and traded. Furthermore if the financial system of a given country brakes-down, due either to exchange rate crisis or to default on government debt, the consequences for the country’s development can be devastating, as a few recent experiences show.

If one examines more closely how financial markets work then one needs to consider the main elements of financial decisions, which are taken either by individual investors or by financial institutions. In both cases portfolio theory relies at the very basis of this decision-making mechanism. Therefore if it is true that there is indeed a significant link between economic development and financial markets, then it follows necessarily that there is a connection between development and portfolio theory.
This paper is divided into eight Sections. In Section 2, the details of the standard macro-models mentioned in the first paragraph are specified. In the following Section, some fundamental issues in Economic Development are examined. The essential aspects of portfolio theory are analyzed in Section 4. Next the macro-models of Section 2 are applied to the American economy. In Section 6, a more elaborate model, which nevertheless has straight-forward links with the simpler ones, is presented. In the following Section, this latter model is applied to the Brazilian economy. Finally, in the last Section, a few results are put forward.

2. “Standard” macro-models

Few areas of economic analysis have been characterized by so large a number of new analytical developments, opposing paradigms, and heated theoretical debates than Macroeconomics. In order to have a “taste” of this state of affairs, one has only to flip through the pages of Macroeconomics textbooks to find, together with new approaches to, e.g., the analysis of open economies, descriptions of the views of Keynesians as opposed to monetarists, or of new-classics in contrast with new-Keynesians.\footnote{See, for example, Froyen (1996).}

However, if someone wishes to concentrate on the bare bones of macro-analysis, especially with an applied or “real-world” approach in mind, then the question emerges of what set of analytical tools could be considered “standard” in the Macroeconomics field and, moreover, would be suitable for applications of a more practical nature. Clearly numerous answers could be given to this question but, in spite of all other possible candidates, two well-known and relatively simple models will be used as the basis for the macro-analysis in this paper: one associated to Kalecki and Klein, and the other to Keynes.

The first of these models, usually known as Klein Model I, is standard material in Econometrics textbooks.\footnote{See Greene (1997), Maddala (1987), Pindick and Rubinfeld (1981), and Wallis (1979).} In these books, only the estimation procedures of the model are examined and, therefore, its true macroeconomic content is not taken into account. Also the
influence of Kalecki’s work on the celebrated Klein Model is usually overlooked, although Klein himself has set the record straight:

“Many economists will recognize the resemblance between the three-equation model, [and] Kalecki’s model of the business cycle […]”

The Keynes model, on the other hand, is formed by the well-known IS-LM equations together with the less-popular equations that represent the labor market, that is, aggregate production function and equilibrium between real wage and marginal labor productivity. The labor-market equations allow for the determination of the price level given the average nominal wage rate.

In order to start examining these models, or any other macro-model, one initial step that usually helps to understand the model’s structure is to work with a broad classification of the equations used. Usually any relation of macroeconomic nature can be classified into one of three distinct analytical blocks: I. Equilibrium output and aggregate production function; II. Effective demand and the generation of income; and III. Determination of nominal variables and inflation.

Normally the emphasis attributed in a given macro-model to each of these blocks varies. For example, in the Kalecki-Klein model, the process of income generation is treated in more detail, and distributive aspects of the income-generation process are considered. In the Keynes model, on the other hand, there are equations for the determination of the price level, while only real-valued variables are included in the Kalecki-Klein model. In Table 1, the equations included in these simple macro-models, classified according to the three blocks introduced in the previous paragraph, are listed.

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4 Klein (1950), p. 63. The so-called Klein Model I has in fact six equations, but Klein is referring only to the behavioral equations. The other three equations are identities. The business cycle theory mentioned in the quotation is developed in Kalecki (1954).
5 See Allen (1968), Ch. 7.
6 In the Keynes model, the equation for investment (I) shown in Table 1 presents a minor change in relation to the original, IS-LM specification, since it includes an accelerator-type relation.
The main proposition in this paper is that these simple macro-models, as well as other models which are based on their straight-forward structural equations, can be very useful in some applied issues in fields as different as Economic Development and Finance. In the next two Sections some basic elements in these areas are examined.

3. Fundamental issues in economic development

Essentially the field of Economic Development deals with the process of creation of wealth in regions and countries, which has numerous and diverse aspects: economic growth, investment and saving, income distribution, foreign trade, structural constraints, factor endowments, and the role of public and private institutions. The indicator of wealth most often used is the national product – e.g., Gross Domestic Product – per capita and, in fact, rich countries are those which present relatively high levels of GDP per inhabitant, and vice-versa.

One of the most challenging issues in Development is probably to explain the observed change in the pattern of wealth of regions and countries through time – or, in some cases, to identify the factors that inhibit change. Table 2 presents data on GDP per capita for 21 countries in several continents over the last half century, and based on these data one can observe major changes in growth trends either through countries and regions, or through time.

The numbers in Table 2 show that major European nations had significant per capita GDP growth rates from 1949 to 2002, although figures in the last columns reflect important variations in exchange rates in relation to the dollar. This growth trend was even more impressive in the case of Japan, which moved from a position inferior to Brazil in 1949 – which obviously reflected the effects of the War – to become one of the world leaders in per capita income. Other Asian countries, especially South Korea, also presented
outstanding economic growth. On the other hand, the performance of Argentina and the African countries listed in the Table is disappointing, with significant reductions in per capita income.

Most likely part of the explanation for these diverse trends in economic growth relies on different macroeconomic policies pursued in each country and region – dealing with exchange-rates, interest rates, fiscal policy and public investment, wage rate policies, and so on –, although certainly other, non-macroeconomic factors also play a role in the growth patterns. If this is in fact the case, then one can definitely justify the use of macro-models that could simulate and forecast the outcomes of alternative macro-policies, especially in relation to economic growth, even given the fact that this can not be done in a precise fashion.

One aspect of economic development that does not commonly get the necessary attention is the fact that the opportunities of investment in a given country, which are essential to enhance growth, depend on the workings of the capital market, both at the national and international levels. Furthermore the main trends and developments in the capital market are determined in part by individual financial decisions. Therefore there is an important link between the process of selection of financial investments and economic development. In the next Section, the basic elements of financial decision making are explored.

4. Overview of portfolio theory

According to modern financial analysis, investment decisions are never based on individual assets, but on portfolios of securities. In other words, no decision is made about a single financial asset without considering its relations with other securities available in the market. In general terms, two elements of a security are taken into account: its expected rate of return over a given time period in the future, represented by $E(\rho_j)$, and its risk, $\sigma_j$, which is usually defined as the standard deviation of the security’s rate of return.\footnote{The definition of risk as a standard deviation does not apply strictly in the case of short-term government bonds, which are considered risk-free.}
In the selection of portfolios of financial assets there are some statistical and algebraic analyses that, by now, have become well known. Given a vector with the shares of the securities in the portfolio, represented by $\xi$, and another vector with the expected rates of return of the securities, $E(\rho)$, then the expected return rate of a portfolio, $E(\rho_p)$, is defined as (the apostrophe represents transposition of a column-vector):

$$E(\rho_p) = \sum_j \xi_j E(\rho_j) = \xi^t E(\rho) \quad (1)$$

On the other hand, given a matrix with variances and co-variances estimated for the securities’ rates of return, represented by $V$, the risk of a portfolio, $\sigma_p$, is defined as:

$$\sigma_p = \sqrt{\xi^t V \xi} \quad (2)$$

In the process of selecting prospective portfolios the main goal is to identify a locus of efficient portfolios, that is, those with maximum expected return for a given risk level or, alternatively, with minimum $\sigma_p$ for a given $E(\rho_p)$. The determination of these loci is a problem of mathematical optimization, and a small-scale example (with three assets) is illustrated in Figure 1 (the efficient set is located above the dashed line).^9

[Figure 1]

Another kind of efficient frontier, more important in the real world, is obtained when a risk-free asset is introduced in the above analysis (see Note 7). In this case, the nature of the frontier changes and it becomes a straight line, illustrated in Figure 2. The new efficient frontier results from all possible combinations of the risk-free asset and portfolio $A$. In equilibrium, portfolio $A$ is the market portfolio, and the straight line is called the Capital Market Line, from which one can read the price of risk determined by the market.

[Figure 2]

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^8 See Haugen (1997), and Huang and Litzenberger (1988).
^9 Fonseca (2003), Ch. 7 and Appendix 7B.
A slightly different analysis, compared to the one illustrated in Figure 2, arises when another measure of risk is introduced: the famous beta coefficient, which is defined as:

$$\beta_j = \frac{\sigma_{jM}}{\sigma_M^2}$$  \hspace{1cm} (3)

In equation (3), $\sigma_{jM}$ represents the co-variance between the rates of return of security $j$ and the market portfolio, and $\sigma_M^2$ stands for the variance of the market return rate. The efficient frontier derived from the beta risk-measure, which is analogous to the one obtained for the sigma risk-measure, is called the Security Market Line and it has the great advantage of allowing individual securities to be compared with portfolios of financial assets – especially, in equilibrium, both securities and portfolios should be located on the Security Line.

Given the analysis in this Section, we can conclude that financial investment decisions rely upon some fundamental parameters: the interest rate (the return rate of risk-free assets), the expected rates of return of securities, and the securities’ risks. The information used in the real world is usually based on historical data, but there is a major problem in this procedure, that is, the outcomes that really matter are the ones that will happen in the future. Since future developments do not often follow smoothly from trends in the past, there is enormous potential for using macro-models to forecast and simulate variables used in financial decisions. Moreover the rates of return of most assets depend on GDP growth, and since real rates are the ones that really matter, inflation forecasts should also play a role. The use of models for forecasting and simulation in financial investment decisions is particularly important in periods marked by ruptures in relation to past trends.
5. Macro-models applied to the American economy

5.1. Kalecki-Klein model: 1921-41

Series of data for the American economy were constructed by Klein for the variables of his model.\textsuperscript{10} From these data, the parameters of the behavioral equations listed in Table I can be estimated. The estimations, based on the Two-Stage Least Squares Estimator, appear in Table 3.

One of the most important aspects of this model – which, nevertheless, apparently was not perceived by other researchers – is that it reproduces fairly well the historical data. This characteristic is illustrated in Figure 3, which presents results for two endogenous variables: disposable income ($Y_D$), and investment ($I$).\textsuperscript{11} Taking into account the impressive transformations illustrated in Figure 3, the favorable performance of the model as a simulation tool should not be considered trivial matter.

5.2. Keynes model: 1954-84

In order to transform the equations of the Keynes model, listed in Table 1, in an empirical tool it is necessary to simplify the labor market equations (second and last equations). Substituting the equation for $L$ in the expression for $w$, and using a logarithmic function, we get:

\[ w = g(Y) P \rightarrow \ln\left(\frac{w}{P}\right) = \xi_0 + \xi_1 \ln Y + \xi_w \]

\[ (4) \]

The parameters of this model were estimated from data series for the American economy, which appear in Table 4. The original data are from 1953 to 1984, but the last five years

\textsuperscript{10} Klein (1950), Appendix.
\textsuperscript{11} Investment data does not include depreciation (net investment) and, therefore, can assume negative values.
were not used in the estimation procedures – they were employed to gauge the model as a tool for forecasting. As in previous periods, historical values for the exogenous variables were used and solutions were calculated for the endogenous variables, so that they could be compared with known historical data. Estimated parameters appear in Table 5.

[Table 4, Table 5]

The model was solved for the 1954-84 period, which goes beyond the interval used for parameter estimation. In the solution, calculated values for \( Y_t \) were used in \( t+1 \). The results appear in Figure 4. In general, values generated by the model are quite close to historical data, especially in the case of investment – a result which, given the cyclical behavior of this variable, can be considered unexpected. On the other hand, results for the interest rate reveal a major flaw of the Keynes system: this variable is considered endogenous but, in the real world, it is determined in great part by the Central Bank (the FED, in this case). Actually the FED promoted a sharp rise of real interest rates starting in the beginning of the 1980s.

[Figure 4]

6. A macro-model with real and nominal variables (R&NV model)

The strong points of the Kalecki-Klein model are the division of families in income groups, and the inclusion of profits among the variables that determine investment. However this model does not include nominal variables and price changes. On the other hand the Keynes model can be used to describe the time path of the general price level, but does not take into account the distribution of income, especially between wages and profits. Therefore a goal that should be pursued is the development of a model that could combine the strong points of the two previous systems – if possible, without introducing further disadvantages. In this Section one such model will be analyzed.\(^{12}\)

\(^{12}\) An earlier version appears in Fonseca (2001).
The structure of the model can be compared with those examined in Sections 2 and 5, that is, the equations can be classified into the same analytical blocks. In general terms, equations in this model are more elaborated and detailed than those in the previous systems. This results from the fact that they were introduced with the specific purpose of being used in forecasting and simulation applied to both real and nominal variables – in fact, the equations for nominal variables and inflation are the ones that depart further from the systems of Sections 2 and 5. The equations of this model appear in Table 6.

[Table 6]

One major difference in relation to the previous models is that in most equations there is a more detailed distinction between real and nominal values. In particular, all variables related to fiscal policy and the financing of government deficits appear in current values. The objective is to explore the relation between public financing, on one side, and the supply of money and inflation, on the other. Furthermore the variables that determine the distribution of income also appear in current values. The goal in this case is to take into account the effects of price and wage changes on income distribution.

Another difference in the model of Table 6 is that the equilibrium equation for national product represents the equality between aggregate income and total output – in this case the sum of production in three sectors: agriculture, industry and services. This specification has the advantage of solving the problem of dealing adequately with the aggregate production function. It permits the determination of the labor force effectively employed using labor coefficients – which are relations in the Leontief, or input-output, tradition.

Equations in the third block of Table 6 are used to describe how fiscal policy and government financing affect monetary aggregates and the time path of prices and wages. In the model, the role of these variables on income distribution, demand and production are also taken into account. The first equation in this block specifies that $M_1$ is determined from the monetary base, given the $M_1$ multiplier ($\mu$). The monetary base, in turn, is a share $\alpha$ of a fraction of broad money ($\beta M_2$) – this fraction is equal to the sum of the monetary base and the public debt. The third equation in the block derives from the government budget
constraint, and the next two equations contribute to determine the time paths of major money aggregates.

In the model, the stock of broad money results directly from the government fiscal deficit (or surplus) and the amount of international reserves and – using coefficients $\alpha$, $\beta$ and the real interest rate $r$ – public debt and the monetary base are calculated. The main reason for this specification is that in many countries, and especially in Latin America, broad money indicators are closely related to the government debt. Furthermore one observes that changes in the evaluation made by financial-market agents about the expected return rate and risk of public bonds, which may also include the possibility of default, can initiate a process of “search for liquidity” – through the selling of outstanding public debt –, which in the end causes increases in the monetary base and $M_1$.\(^{13}\)

The last two equations in the third block describe the time path of prices and wages. The main hypothesis used is that price changes result from two components, one determined by demand factors – the change of $M_1$ (times its income-velocity, represented by $V$) in excess of real output variation – and the other by cost factors (changes in wages, basic input prices, and the exchange rate).\(^{14}\) The estimation procedures for the model equations and the data series used are described in Fonseca (2000).

7. The R&NV model applied to the Brazilian economy: 1973-2010

The model analyzed in the previous Section was estimated and solved using data for Brazil. In the solution, the Gauss-Seidl method was employed – the necessary codes were written in Microsoft Visual Basic. Data for the 1968-98 period were used in the estimation procedures, while the solution was calculated for the years from 1973 to 2010. Results for some of the model’s endogenous variables appear in Figure 5.

\[\text{Figure 5}\]

\(^{13}\) In the model, this process would correspond to changes in the coefficient $\alpha$.

In general, the model reproduces the main tendencies of the Brazilian economy in the last decades: the historical trend of GDP, the pattern of income distribution between wages and profits, the time path of public debt and money supply, and the inflation trend. During the 1985-95 period government expenditures, especially with wages and salaries, rose in an impressive fashion – the share of government wage spending in GDP increased from 6.7 to 14.3%. However, quite differently from what one would expect based on Keynes’ model, real GDP did not increase significantly and, moreover, the decades of 1980 and 1990 were marked by stagnation of investment. These developments were also reproduced in the model’s solutions.

From 2004 onwards, GDP data were not available at the time this paper was written (beginning of 2005) and solutions were obtained basically from assumptions for the future path of exogenous variables and some key coefficients – like $M_1$ income-velocity ($V$), $M_1$ multiplier ($\mu$), the shares of monetary base and government debt on broad money (coefficients $\alpha$ and $\beta$), as well as the dynamic effects of lagged inflation on wage increases (the price-wage indexation mechanism, which is implicit in function $h$ in Table 6).

Figure 5 shows that there have been major changes in macroeconomic trends in Brazil, especially in the government fiscal balance and debt, and in the pattern of inflation. These changes could be effectively reproduced in the model’s solutions due in part to the fact that the coefficients mentioned in the previous paragraph are allowed to vary in the solution procedures. Additionally it is an empirical fact that they have indeed varied quite dramatically in Brazil’s economy. ¹⁵

Probably the most important results illustrated in Figure 5 are that the extension of current (beginning of 2005) trends of Brazil’s economy into the future should bring low economic growth, relatively moderate fiscal deficits and a decreasing public debt – since government revenues, due to ever higher tax rates, have reached record levels –, together with moderately high inflation (for Brazilian standards). These forecasts have implications both for economic development issues and for those involved in financial investment decisions.

¹⁵ Fonseca (2005).
8. Conclusion: macro-models in Development and Finance

One of the main objectives in the field of Development is to identify long-term strategies that would lead to rapid and lasting economic growth in a given country or region. Certainly not all aspects of such strategies are related to macroeconomic policies and, in fact, many social and institutional factors are present in successful experiences in economic development – probably an adequate system of mass education is one of the most important of these factors.

Yet macroeconomic policies that are well conceived and correctly applied certainly play an important role in the development process. However, to identify “adequate” policies, that would bring the desired results in the long-run, is not a simple task and therefore macro-models conceived for simulation and forecasting purposes can definitely play a role in the process of devising strategies to achieve some agreed-upon goals – something which is often called economic planning. The models examined in the previous Sections, with very satisfactory results in terms of simulation and prediction, are examples of tools that can be applied to planning, not only in the public sector but also as part of the process of specifying strategic goals and programs in the private sector.

In the area of Finance the main goal is quite different from those in Development – simply to obtain the highest possible rate of return for a given level of risk. Yet it was shown above that the variables that affect both the decision-making process and the ultimate results that follow from these decisions are linked to macroeconomic variables. Therefore macro-models can also be used in the process of selecting financial investments. The application of macro-models in Finance is more valid when the investment decisions take into account the long-run. Actually there are several kinds of financial institutions, like insurance companies and private pension-plan institutions, in which decisions are taken with longer periods in view.
References


Table 1
Equations in the Kalecki-Klein and Keynes Models

<table>
<thead>
<tr>
<th></th>
<th>Kalecki-Klein</th>
<th>Keynes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equilibrium output and</td>
<td>$Y^D + T = C + I + G + (EX - IM)$</td>
<td>$Y = C + I + G + (EX - IM)$</td>
</tr>
<tr>
<td>aggregate production function</td>
<td>$L = f^{-1}(Y)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Effective demand and</td>
<td>$C = \alpha_0 + \alpha_1 \Pi + \alpha_2 (W_1 + W_2) + \alpha_3 \Pi_{-1} + \varepsilon_C$</td>
<td>$C = \alpha_0 + \alpha_1 (Y - T) + \varepsilon_C$</td>
</tr>
<tr>
<td>the generation of income</td>
<td>$I = \beta_0 + \beta_1 \Pi + \beta_2 \Pi_{-1} + \beta_3 K_{-1} + \varepsilon_I$</td>
<td>$I = \beta_0 + \beta_1 r + \beta_2 \Delta Y + \beta_3 \gamma_{-1} + \varepsilon_I$</td>
</tr>
<tr>
<td></td>
<td>$W_1 = \gamma_0 + \gamma_1 (Y^D + T - W_2) + \gamma_2 (Y^D + T - W_2)_{-1} + \gamma_3 (t - 1931) + \varepsilon_w$</td>
<td>$\frac{M}{P} = \gamma_0 + \gamma_1 r + \gamma_2 Y + \varepsilon_M$</td>
</tr>
<tr>
<td></td>
<td>$\Pi = Y^D - W_1 - W_2$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$K = K_{-1} + I$</td>
<td></td>
</tr>
<tr>
<td>Determination of</td>
<td>$w = \left[ \frac{d}{dL} f \right] P$</td>
<td></td>
</tr>
<tr>
<td>nominal variables and</td>
<td></td>
<td></td>
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<tr>
<td>inflation</td>
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</tbody>
</table>

Description of the variables:
$Y$: National product; $Y^D$: Disposable income; $T$: Taxes; $C$: Private consumption; $I$: Aggregate investment; $G$: Government consumption; $(EX-IM)$: Trade balance; $L$: Employed labor force; $r$: Interest rate (real rate); $M$: Money supply; $P$: National product deflator; $W_1$: Total wages in the private sector; $W_2$: Total wages in the public sector; $\Pi$: Profits; $w$: average nominal wage rate; $K$: Capital stock; $t$: time (years); $\varepsilon_i$: errors (random variables).


Exogenous variables in the Keynes model: $T, G, (EX-IM), M, w$. 
### Table 2
National Product Per Capita (Real dollars in 2002 values)\(^1\)

<table>
<thead>
<tr>
<th>Year</th>
<th>United States</th>
<th>Japan</th>
<th>Great Britain</th>
<th>Germany</th>
<th>Canada</th>
<th>France</th>
<th>Italy</th>
<th>Sweden</th>
<th>Australia</th>
<th>Spain</th>
<th>Mexico</th>
<th>Chile</th>
<th>Venezuela</th>
<th>Argentina</th>
<th>Brazil</th>
<th>South Korea</th>
<th>Thailand</th>
<th>China</th>
<th>India</th>
<th>Nigeria</th>
<th>Ethiopia</th>
</tr>
</thead>
<tbody>
<tr>
<td>1949</td>
<td>13,916</td>
<td>804</td>
<td>6,214</td>
<td>2,573</td>
<td>6,994</td>
<td>3,875</td>
<td>1,889</td>
<td>6,271</td>
<td>5,459</td>
<td>n.a.</td>
<td>973</td>
<td>1,511</td>
<td>2,589</td>
<td>2,782</td>
<td>900</td>
<td>n.a.</td>
<td>289</td>
<td>217</td>
<td>458</td>
<td>444</td>
<td>305</td>
</tr>
<tr>
<td>1953</td>
<td>16,468</td>
<td>1,412</td>
<td>6,664</td>
<td>3,454</td>
<td>9,445</td>
<td>4,300</td>
<td>2,200</td>
<td>6,521</td>
<td>6,600</td>
<td>1,734</td>
<td>1,433</td>
<td>1,792</td>
<td>3,798</td>
<td>2,623</td>
<td>1,541</td>
<td>502</td>
<td>545</td>
<td>358</td>
<td>430</td>
<td>504</td>
<td>358</td>
</tr>
<tr>
<td>1965</td>
<td>21,606</td>
<td>5,206</td>
<td>10,958</td>
<td>11,122</td>
<td>14,967</td>
<td>11,687</td>
<td>6,688</td>
<td>15,167</td>
<td>12,203</td>
<td>14,237</td>
<td>2,764</td>
<td>n.a.</td>
<td>n.a.</td>
<td>2,988</td>
<td>1,701</td>
<td>592</td>
<td>765</td>
<td>595</td>
<td>601</td>
<td>304</td>
<td>128</td>
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<tr>
<td>1980</td>
<td>20,075</td>
<td>20,389</td>
<td>13,438</td>
<td>17,982</td>
<td>16,668</td>
<td>18,297</td>
<td>15,143</td>
<td>20,916</td>
<td>14,237</td>
<td>21,840</td>
<td>3,853</td>
<td>2,002</td>
<td>3,112</td>
<td>7,706</td>
<td>2,818</td>
<td>2,750</td>
<td>942</td>
<td>176</td>
<td>204</td>
<td>304</td>
<td>374</td>
</tr>
<tr>
<td>2002</td>
<td>35,060</td>
<td>33,550</td>
<td>25,250</td>
<td>22,670</td>
<td>22,300</td>
<td>22,010</td>
<td>18,960</td>
<td>30,876</td>
<td>21,840</td>
<td>19,740</td>
<td>5,910</td>
<td>4,260</td>
<td>4,090</td>
<td>4,060</td>
<td>2,850</td>
<td>9,930</td>
<td>1,980</td>
<td>940</td>
<td>480</td>
<td>290</td>
<td>100</td>
</tr>
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</table>

Notes:
1. Data in constant values calculated by the author based on the US Consumer Price Index.
7. In 1949 e 1953, data for West Germany. From 1965 on, numbers correspond to modern Germany.

n.a. – Information is not available.
Table 3  
*Estimation of the Equations in the Kalecki-Klein Model*  
*(Two-Stage Least Squares Estimator)*

<table>
<thead>
<tr>
<th>Dependent Variable: $C$</th>
<th>Independent Vars.:</th>
<th>Coefficient</th>
<th>Standard Error</th>
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<td>$\Pi_{-1}$</td>
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<td>0.1073</td>
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<table>
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<th>Coefficient</th>
<th>Standard Error</th>
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<tbody>
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<td></td>
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<td></td>
<td>$\Pi_{-1}$</td>
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<td>$K_{-1}$</td>
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<table>
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<th>Dependent Variable: $W_1$</th>
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<th>Coefficient</th>
<th>Standard Error</th>
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<td>$(\gamma + T - W_2)_1$</td>
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<td>$\tau$–1931</td>
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<td>0.0291</td>
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Note:  
All variables in billions of dollars of 1934 (constant units), $\tau$ in years. Number of observations: 21 (1921–41).

Sources:  
Data source is Klein (1950), Appendix. Estimates were calculated by the author of this paper.
Table 4
Data Series Used in the Keynes Model

<table>
<thead>
<tr>
<th>Year</th>
<th>Y</th>
<th>C</th>
<th>I</th>
<th>G</th>
<th>EX-IM</th>
<th>Y-T</th>
<th>r^2</th>
<th>P^3</th>
<th>M/P^4</th>
<th>w/P^5</th>
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<td>85.3</td>
<td>170.1</td>
<td>4.8</td>
<td>399.1</td>
<td>1.62</td>
<td>58.82</td>
<td>126</td>
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<td>156</td>
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<td>403.6</td>
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<td>223.44</td>
<td>547.3</td>
<td>209.09</td>
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</table>

Sources:
Baumol and Blinder (1985), and Greene (1997).

Notes:
1. Variables in billions of dollars of 1972, r in %, P and w are indices (1972=100).
2. Average interest rates of corporate bonds with Moody’s Aaa rating.
3. GNP deflator.
4. M1 stock.
5. Real average hourly earnings.
Table 5
Estimation of the Equations in the Keynes Model
(Two-Stage Least Squares Estimator)

<table>
<thead>
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<th>Dependent Variable: $C$</th>
<th>Independent Vars.:</th>
<th>$Y-T$</th>
</tr>
</thead>
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<tr>
<td>Coefficient</td>
<td>8.052</td>
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<td>Standard Error</td>
<td>5.297</td>
<td>0.0076</td>
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<table>
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<th>Dependent Variable: $I$</th>
<th>Independent Vars.:</th>
<th>$r$</th>
<th>$\Delta Y$</th>
<th>$Y_{-1}$</th>
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<tr>
<td>Coefficient</td>
<td>-6.437</td>
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</table>

<table>
<thead>
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<th>Dependent Variable: $r$</th>
<th>Independent Vars.:</th>
<th>$M/P$</th>
<th>$Y$</th>
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<td>Coefficient</td>
<td>1.1619</td>
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<td>0.00423</td>
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<tr>
<td>Standard Error</td>
<td>0.9955</td>
<td>0.00978</td>
<td>0.00277</td>
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<table>
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<th>Dependent Variable: $\ln(w/P)$</th>
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<th>$\ln Y$</th>
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<td>Standard Error</td>
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<td>0.0226</td>
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</table>

Note:
Variables in billions of dollars of 1972, $r$ in %, $P$ and $w$ are indices (1972=100). Number of observations: 26 (1954-79).

Sources:
For original data sources, see Table 4. Estimates were calculated by the author.
Table 6
Equations in the Model with Real and Nominal Values
(R&NV model)

| Equilibrium output and aggregate production function | \( Y = Y_{AGR} + Y_{IND} + Y_{SERV} + \frac{1}{P} (\text{Indirect Taxes} - \text{Subsidies}) \)  
| \( L = a_1 Y_{AGR} + a_2 Y_{IND} + a_3 Y_{SERV} \) |
| Effective demand and the generation of income | \( Y_{IND} = \begin{bmatrix} Y \left[ \frac{1}{P} (w L - T_W), \frac{1}{P} (\Pi - T_{\Pi}), I, \frac{1}{P} G, \frac{e}{P} (EX - IM) \right] \end{bmatrix} \)  
| \( I = I \left[ r, \frac{1}{P} \Pi, \Delta Y_{-1} \right] \)  
| \( \Pi = YP - wL - \text{Indirect Taxes} + \text{Subsidies} \) |
| Determination of nominal variables and Inflation | \( M_1 = \mu B \)  
| \( B = \alpha (\beta M_2) \)  
| \( \Delta (\beta M_2) = G + \text{Interest} + \text{Subsidies} - T_W - T_{\Pi} - \text{Indirect Taxes} + e (\Delta \text{Reserves}) \)  
| \( \text{Interest} = \left[ \frac{P}{P_{-t}} (1 + r) - 1 \right] \text{Debt}_{-t} \)  
| \( \text{Debt} = (1 - \alpha) (\beta M_2) \)  
| \( \Delta \ln P = g [\Delta \ln (M_1 V) - \Delta \ln (Y), \Delta \ln (w), \Delta \ln (\text{Inputs}), \Delta \ln (e)] \)  
| \( \Delta \ln (w) = h [\Delta \ln (P_{-1})] \) |

Description of the variables:

\( Y \): Aggregate income and product, in real values; \( Y_{AGR} \): GDP of the agricultural sector, in real values; \( Y_{IND} \): GDP of the industrial sector, in real values; \( Y_{SERV} \): GDP of the services sector, in real values; \( L \): Total employment (number of persons); \( I \): Investment, in real values; \( r \): Interest rate, in real terms; \( P \): Average price level; \( G \): Government spending, in current values; \( EX-IM \): Trade balance, in current dollars; \( e \): Average exchange rate; \( M_1 \): Money stock, current values; \( w \): Average wage rate, current values; \( T_W \): Taxes on wages and salaries, in current values; \( T_{\Pi} \): Taxes on profits, in current values; \( \Pi \): Total profits, in current values; \( B \): Monetary base, current values; \( M_2 \): \( M_1 \) + time deposits + money market securities + government securities not owned by financial institutions, current values; \( \text{Interest} \): Interest payments on the public debt, current values; \( \text{Reserves} \): International reserves, current dollars; \( \text{Debt} \): Public debt, current values; \( \text{Inputs} \): Average price of basic commodities (oil, raw materials, essential agricultural products, public utilities), current values.

Exogenous variables:

\( Y_{AGR}, \text{Indirect Taxes, Subsidies, (EX-IM), } T_W, T_{\Pi}, r, \text{Reserves, } e, \text{Inputs} \).
Figure 1
Example of an Efficient Frontier of Portfolios of Securities

Data for three securities:
\[ E(\rho_1) = 14, \ E(\rho_2) = 11, \ E(\rho_3) = 16, \ \sigma_1 = 18, \ \sigma_2 = 16, \ \sigma_3 = 30, \ \sigma_{12} = 28,8, \ \sigma_{13} = 81, \ \sigma_{23} = 192 \]

Equation of the hyperbole:
\[ \sigma_\rho^2 = 23,59[E(\rho_\rho) - 12,38]^2 + 156,94 \]
Figure 2

Efficient Frontier of Portfolios of Securities with Risk-Free Asset

Data for the risk-free asset:

\[ \rho_f = 5 \]

Equation of the straight line:

Portfolio A: \[ \xi_1 = 0,526, \xi_2 = 0,333, \xi_3 = 0,141 \]

Line: \[ E(\rho_p) = 5 + 0,624 \sigma_p \]
Figure 3

Macroeconomic Data for the United States: 1921-41
(Kalecki-Klein Model)
A. Disposable income; B. Net investment

Sources:
Historical data – Klein (1950), Appendix. Simulation – model’s solutions calculated by the author of this paper.
Figure 4

Macroeconomic Data for the United States: 1954-84

(Keynes Model)
Figure 4 (cont.)

- Série histórica
- Simulação
Figure 5
Macroeconomic Data for Brazil: 1970-2010
(R&NV Model)

Notes:
Real data in billions of 1980 cruzeiros; shares in %; total employment in thousands; price changes in logarithmic differences. Variables defined in Table 6 (Ygdp: GDP in real values, Ynom: GDP in nominal values, Ipriv: private investment).
Figure 5 (cont.)
Figure 5 (cont.)

**Total employment**

- **Historical data**
- **Simulated**

**G_surplus/Ynom**

- **Historical data**
- **Simulated**

**G_debt/Ynom**

- **Historical data**
- **Simulated**
Figure 5 (cont.)

Monetary_base/Ynom

\[ \begin{array}{c|ccccccccccccccc}
\hline
\text{Historical data} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
\text{Simulated} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
\end{array} \]

M/Ynom

\[ \begin{array}{c|ccccccccccccccc}
\hline
\text{Historical data} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
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\end{array} \]

Change in \( P \)

\[ \begin{array}{c|ccccccccccccccc}
\hline
\text{Historical data} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
\text{Simulated} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
\end{array} \]